Frame based kernel methods for hyperspectral imagery data

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Acknowledgements and collaborators

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Collaborators:

- John J. Benedetto, University of Maryland, College Park.
- Wojciech Czaja, University of Maryland, College Park.
- J. Christopher Flake, University of Maryland, College Park.

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Outline

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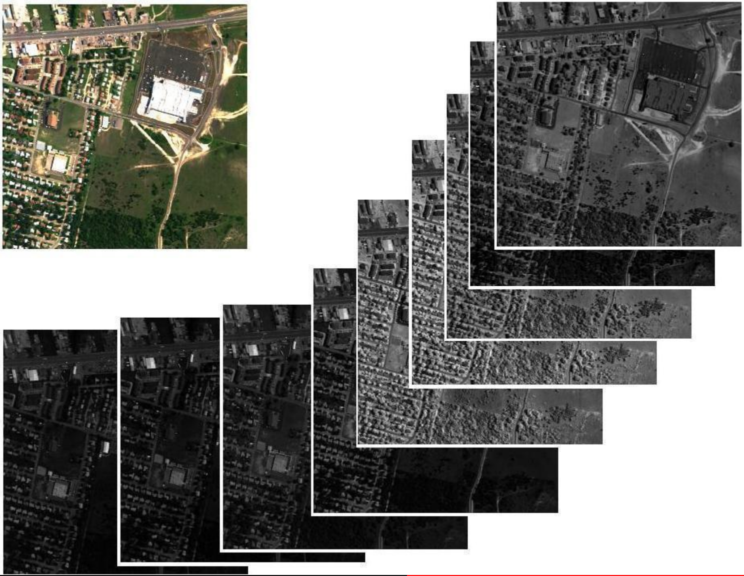
Color image

Red

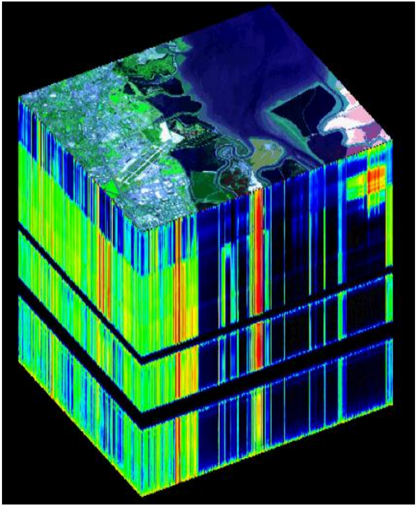
Blue

Green

Hyperspectral imagery data



Hyperspectral data cube



Overview of hyperspectral imagery data

- Hyperspectral imagery (HSI) data is characterized by the narrowness and contiguous nature of the measurements.
- HSI data sets are spectrally overdetermined, and thus provide ample spectral information to distinguish between spectrally similar (but unique) materials.
- HSI data sets can be useful for the following purposes:
  - target detection
  - material classification
  - material identification
  - mapping details of surface properties

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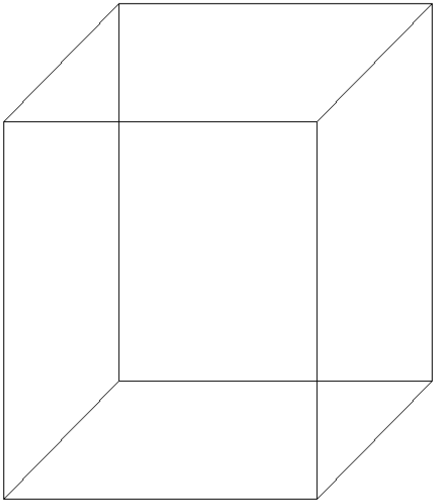
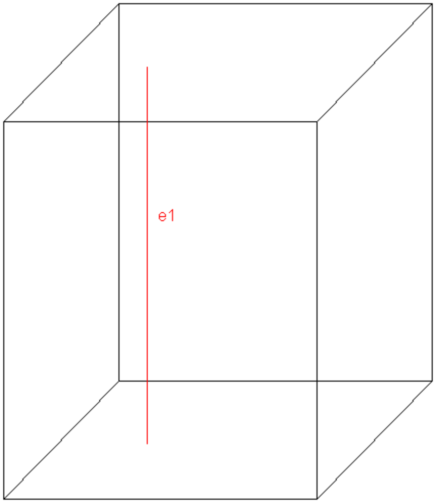
Endmember illustration



Endmember illustration



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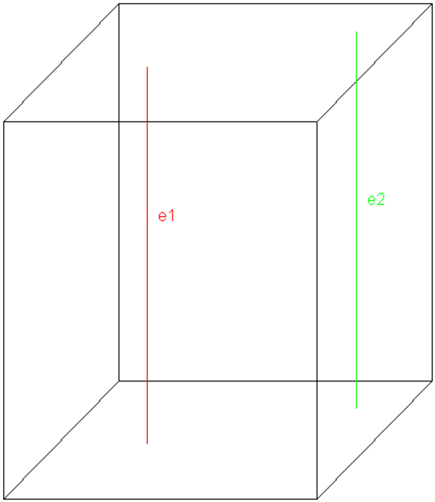
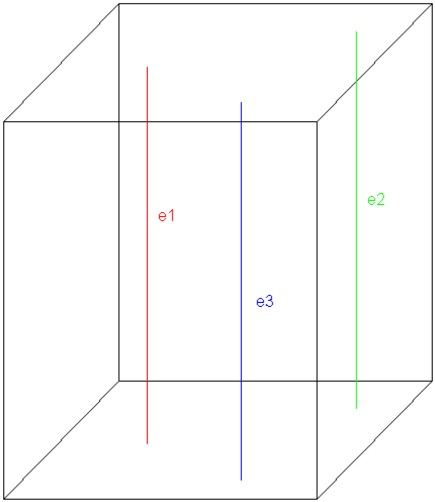
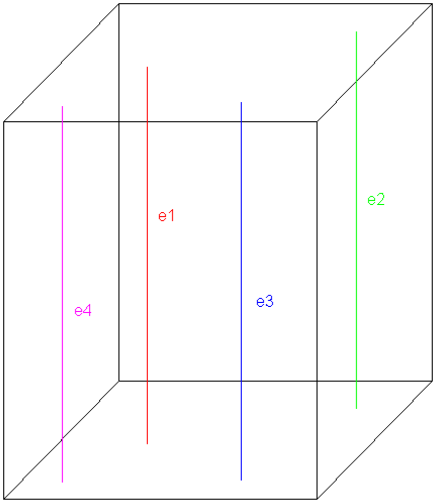
Endmember illustration



Endmember illustration



Endmember illustration



Endmember definition

- Assume our HSI data set is an $N_1 \times N_2 \times D$ cube.
  - N_1, N_2 spatial dimensions; $N = N_1 N_2$ pixels.
  - D is the spectral dimension; so D wavelengths measured.
- Let $X = \{x_i\}_{i=1}^{N} \subset \mathbb{R}^D$ denote the pixel vectors of the HSI data set in list form.

Definition

Endmembers are a collection of a scene's constituent spectra. If $E = \{e_i\}_{i=1}^{s}$ are endmembers, then the linear mixture model is

$$x_i = \sum_{j=1}^{s} \alpha_{i,j} e_j + N_{x_i}, \quad \forall\, x_i \in X,$$

where N_{x_i} is a noise vector.

Outline

1. Hyperspectral imagery data
   - Introduction to hyperspectral imagery data
   - Endmembers
2. The algorithm
   - Kernel eigenmap methods
   - Frames
3. Results
   - The Urban data set
   - Classification results

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Introduction to the algorithm

We have two goals:

1. Map the high dimensional HSI data set X to an appropriate low dimensional space Y.
2. Represent the low dimensional space Y for the purposes of material classification.

We achieve these goals via two existing mathematical theories:

1. We use kernel eigenmap methods to determine the space Y.
2. We represent Y with an overcomplete endmember set Φ, also known as a frame.

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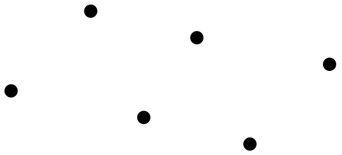
Introduction to kernel eigenmap methods

- Given a high dimensional data set $X = \{x_i\}_{i=1}^N \subset \mathbb{R}^D$, we assume X lies on a low dimensional manifold M^d $(d < D)$.
- Kernel eigenmap methods map the vectors in X to d-dimensional coordinates $Y = \{y_i\}_{i=1}^N \subset \mathbb{R}^d$ that preserve the underlying geometry of M^d.



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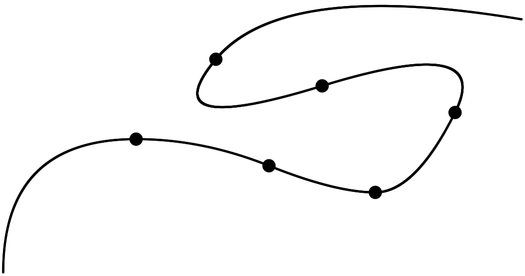

Basics of kernel eigenmap methods

The main components of kernel eigenmap methods are:

- Construction of an $N \times N$ symmetric, positive semi-definite kernel K,

$$K_{i,j} = K(x_i, x_j).$$

- Diagonalization of K and then choosing $d \leq D$ significant orthogonal eigenvectors of K, denoted by $v_1, \ldots, v_d$.
- Map each $x_i \in X$ to the d-dimensional vector y_i given by:

$$y_i = (v_1(i), \ldots, v_d(i)).$$

Frame theory

Definition

1. $\Phi = \{\varphi_i\}_{i=1}^{s}$ is a frame for $\mathbb{R}^d$ if there exists $A, B > 0$ such that

$$A\|y\|^2 \leq \sum_{i=1}^{s} |\langle y, \varphi_i \rangle|^2 \leq B\|y\|^2, \quad \forall\, y \in \mathbb{R}^d.$$

2. For a frame $\Phi = \{\varphi_i\}_{i=1}^{s}$, the frame operator $S : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ is

$$S(y) = \sum_{i=1}^{s} \langle y, \varphi_i \rangle \varphi_i.$$

Frames and HSI data

- We wish to use a data dependent frame $\Phi = \{\varphi_i\}_{i=1}^{s}$ to represent the reduced dimension space $Y = \{y_i\}_{i=1}^{N} \subset \mathbb{R}^d$:

$$y_i = \sum_{j=1}^{s} c_{i,j}\varphi_j, \quad \forall\, y_i \in Y.$$

- Possible frame construction algorithms:
  - existing endmember algorithms
  - modified frame potential [Benedetto, Fickus; 2003]
- Possible frame coefficients:
  - $c_{i,j} = \langle y_i, S^{-1}(\varphi_j)\rangle$
  - $c_{i,\cdot} = \arg\min_c \|c\|_{\ell^1}$ subject to $\Phi c = y_i$

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Why use frames?

Why use frames?

- Traditional endmembers may be mixtures of many prominent features.
- Overestimating the number of classes allows for flexibility in representing mixtures and pure elements.
- Empirical evidence suggests that distinct classes are not orthogonal to each other. Unlike the orthogonal eigenvectors of K, frame elements need not be orthogonal.

Review of algorithm

$$
\begin{tikzcd}
 & K \subset \mathbb{R}^N \arrow[ddd] & \\
X \subset \mathbb{R}^D \arrow[ur] & & \\
 & & \{c_{i,j}\} \subset \mathbb{R}^s \\
 & Y \subset \mathbb{R}^d \arrow[ur, "\Phi"'] &
\end{tikzcd}
$$

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Urban

Figure: HYDICE Copperas Cove, TX – http://www.tec.army.mil/Hypercube/

Urban classes

Figure: HYDICE Copperas Cove, TX – http://www.tec.army.mil/Hypercube/

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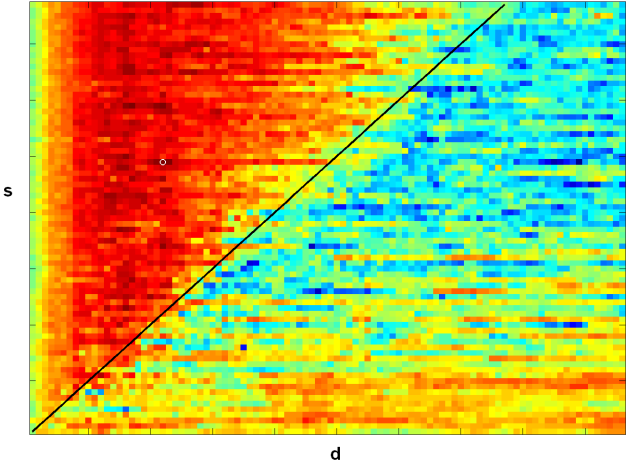
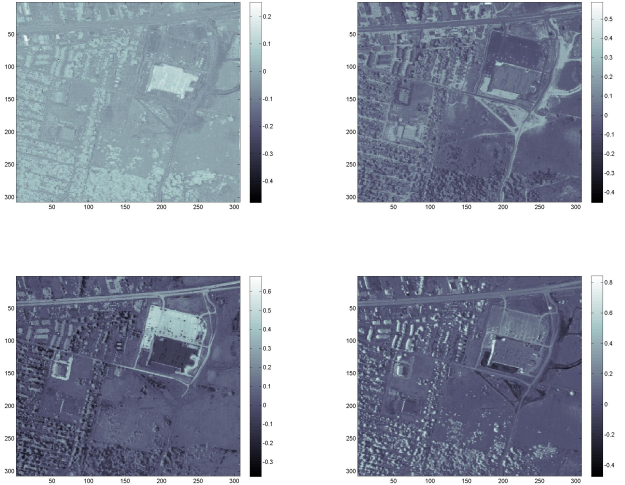
Overview of classification results

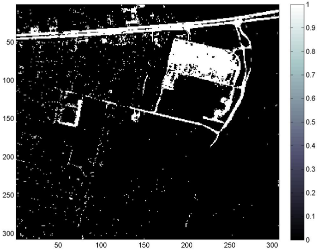


Sample frame coefficients



Dark asphalt

(e) Urban



(f) Dark Asphalt

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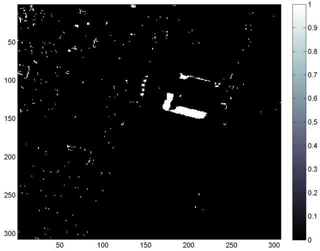
Light asphalt

(a) Urban

(b) Light asphalt

Concrete

(a) Urban



(b) Concrete

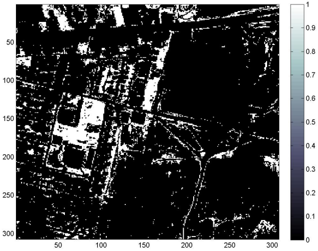
Vegetation: pasture

(a) Urban

(b) Vegetation: pasture

Vegetation: grass

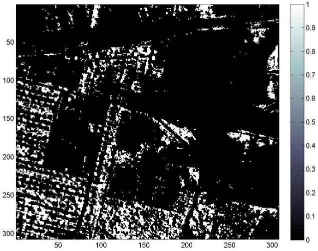
(a) Urban



(b) Vegetation: grass

Vegetation: trees

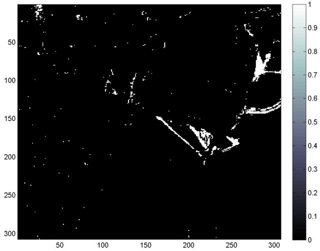
(a) Urban



(b) Vegetation: trees

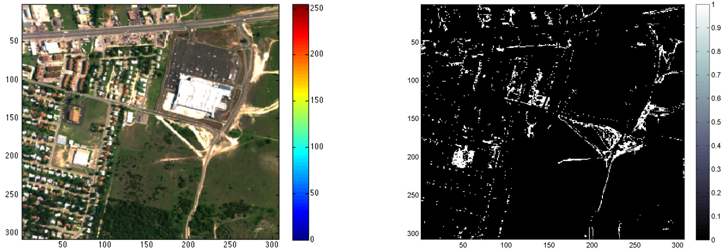
Soil 1

(a) Urban



(b) Soil 1

Soil 2



(a) Urban (b) Soil 2

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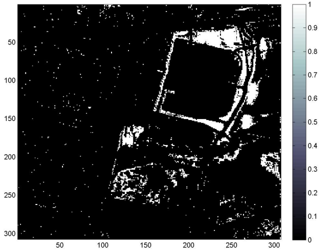
Soil 3 (dark)

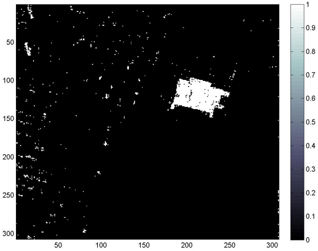
(a) Urban



(b) Soil 3 (dark)

Roof: Walmart

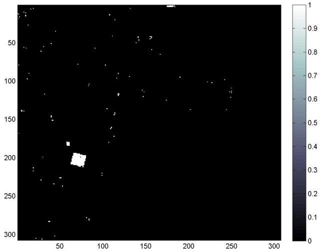
(a) Urban



(b) Roof: Walmart

Roof: A

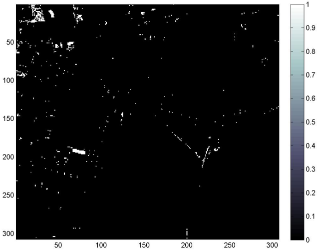
(a) Urban



(b) Roof: A

Roof: B

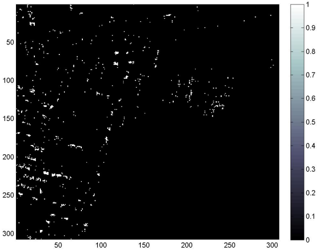
(a) Urban



(b) Roof: B

Roof: light gray

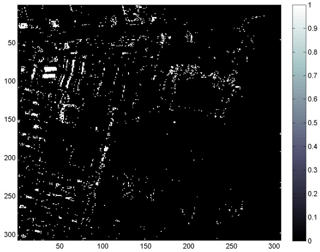
(a) Urban



(b) Roof: light gray

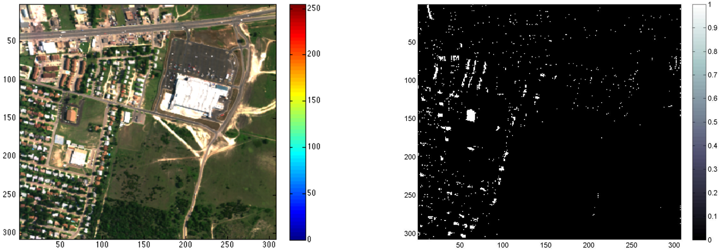
Roof: dark brown

(a) Urban



(b) Roof: dark brown

Roof: church

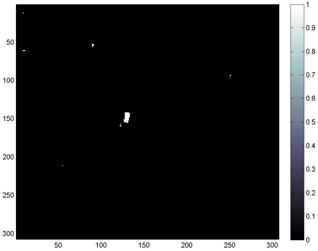


(a) Urban

(b) Roof: church

Roof: school

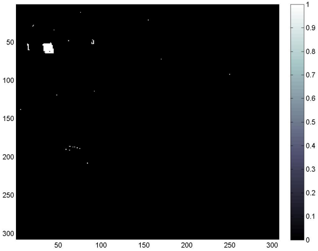
(a) Urban



(b) Roof: school

Roof: bright

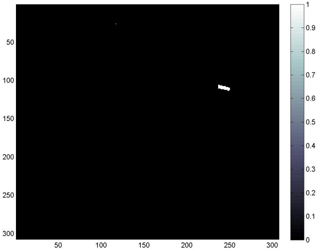
(a) Urban



(b) Roof: bright

Roof: blue green

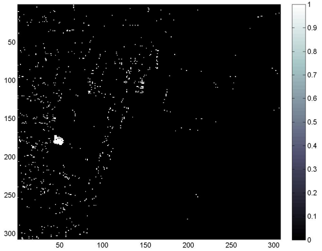
(a) Urban



(b) Roof: blue green

Tennis court

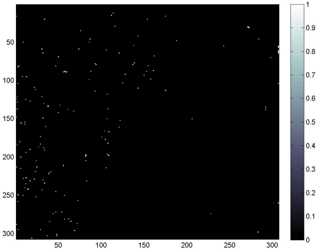
(a) Urban



(b) Tennis court

Pool water

(a) Urban



(b) Pool water

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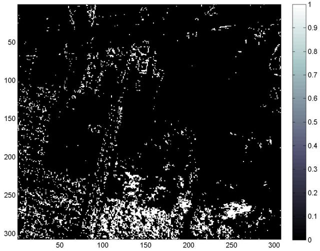
Shaded vegetation

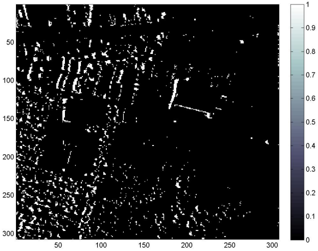
(a) Urban



(b) Shaded vegetation

Shaded pavement

(a) Urban



(b) Shaded pavement

Thank you

Thank you for your time.

http://www.math.umd.edu/∼hirn/