Frame based kernel methods for hyperspectral imagery data

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Recent Advances in Harmonic Analysis
and Elliptic Partial Differential Equations

University of Virginia

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Acknowledgements and collaborators

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- PI: John J. Benedetto, University of Maryland, College Park.

Collaborators:

- John J. Benedetto, University of Maryland, College Park.
- Wojciech Czaja, University of Maryland, College Park.
- J. Christopher Flake, University of Maryland, College Park.

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Outline

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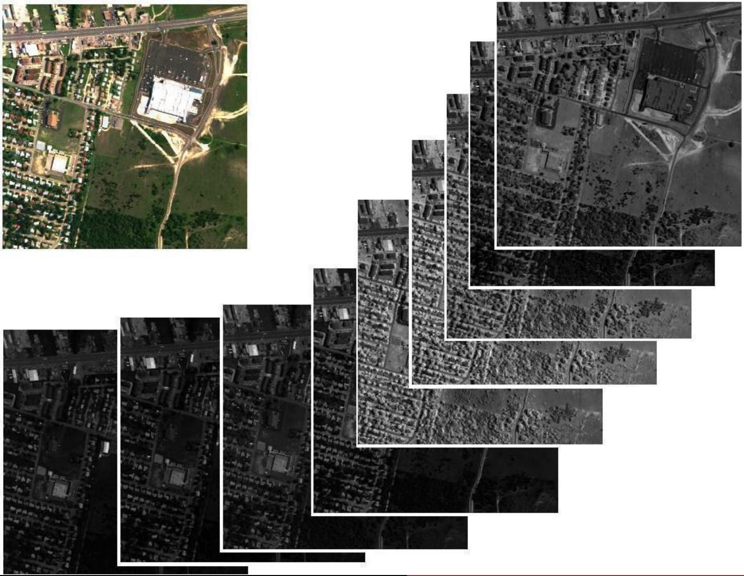
Color image

Red

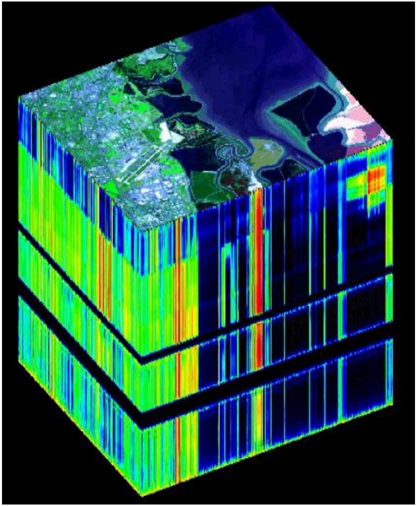
Blue

Green

Hyperspectral imagery data



Hyperspectral data cube



Overview of hyperspectral imagery data

- Hyperspectral imagery (HSI) data is characterized by the narrowness and contiguous nature of the measurements.
- HSI data sets are spectrally overdetermined, and thus provide ample spectral information to distinguish between spectrally similar (but unique) materials.
- HSI data sets can be useful for the following purposes:
  - target detection
  - material classification
  - material identification
  - mapping details of surface properties

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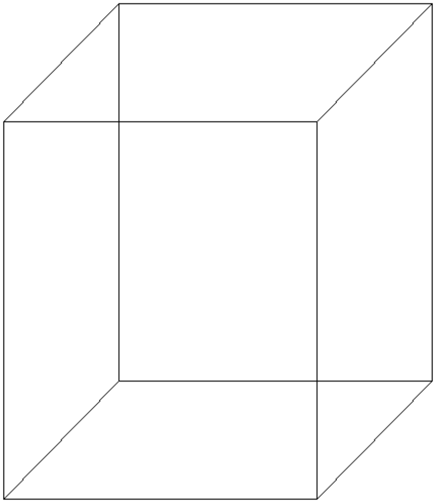
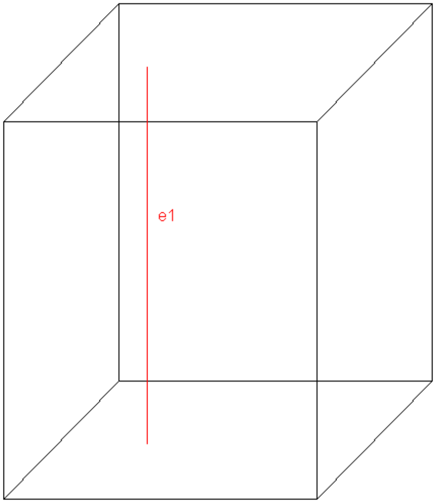
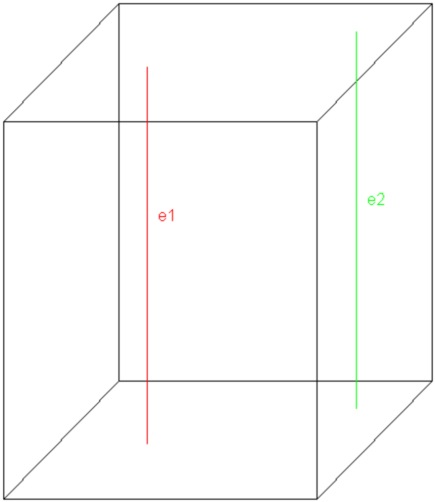
Endmember illustration



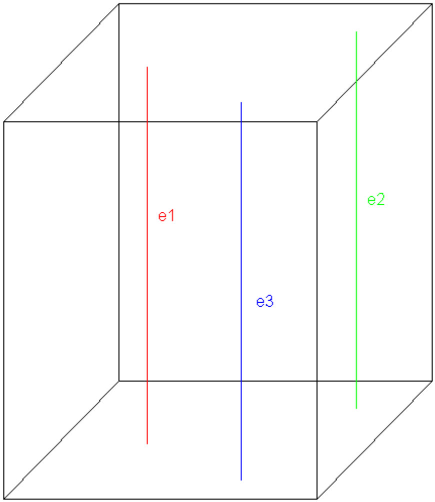
Endmember illustration



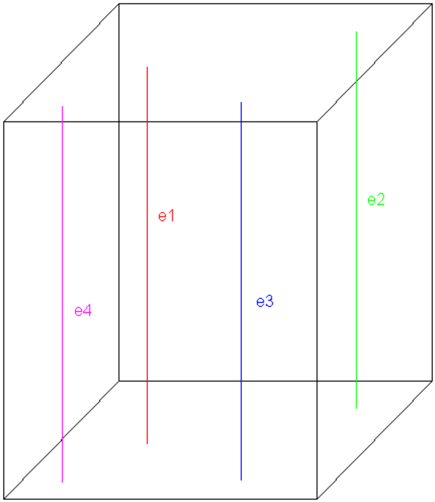
Endmember illustration



Endmember illustration



Endmember illustration



Endmember definition

- Assume our HSI data set is an $N_1 \times N_2 \times D$ cube.
  - N_1, N_2 spatial dimensions; $N = N_1 N_2$ pixels.
  - D is the spectral dimension; so D wavelengths measured.
- Let $X = \{x_i\}_{i=1}^N \subset \mathbb{R}^D$ denote the pixel vectors of the HSI data set in list form.

Definition

Endmembers are a collection of a scene's constituent spectra. If $E = \{e_i\}_{i=1}^s$ are endmembers, then the linear mixture model is

$$x_i = \sum_{j=1}^{s} \alpha_{i,j} e_j + N_{x_i}, \quad \forall\, x_i \in X,$$

where N_{x_i} is a noise vector.

Outline

Introduction to the algorithm

We have two goals:

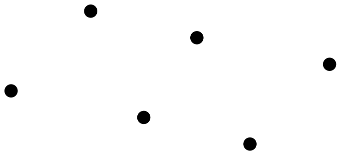
1. Map the high dimensional HSI data set X to an appropriate low dimensional space Y.
2. Represent the low dimensional space Y for the purposes of material classification.

We achieve these goals via two existing mathematical theories:

1. We use kernel eigenmap methods to determine the space Y.
2. We represent Y with an overcomplete endmember set Φ, also known as a frame.

Introduction to kernel eigenmap methods

- Given a high dimensional data set $X = \{x_i\}_{i=1}^N \subset \mathbb{R}^D$, we assume X lies on a low dimensional manifold M^d $(d < D)$.
- Kernel eigenmap methods map the vectors in X to d-dimensional coordinates $Y = \{y_i\}_{i=1}^N \subset \mathbb{R}^d$ that preserve the underlying geometry of M^d.



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Introduction to kernel eigenmap methods

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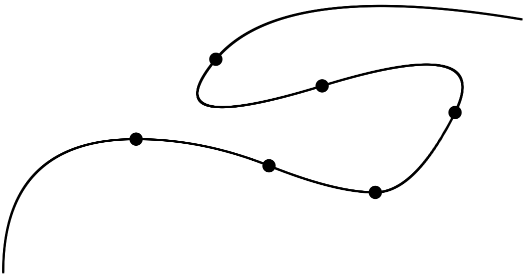

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Basics of kernel eigenmap methods

The main components of kernel eigenmap methods are:

- Construction of an $N \times N$ symmetric, positive semi-definite kernel K,
$$K_{i,j} = K(x_i, x_j).$$
- Diagonalization of K and then choosing $d \leq D$ significant orthogonal eigenvectors of K, denoted by $v_1, \ldots, v_d$.
- Map each $x_i \in X$ to the d-dimensional vector y_i given by:
$$y_i = (v_1(i), \ldots, v_d(i)).$$

Frame theory

Definition

1. $\Phi = \{\varphi_i\}_{i=1}^s$ is a frame for $\mathbb{R}^d$ if there exists $A, B > 0$ such that

$$A\|y\|^2 \le \sum_{i=1}^{s} |\langle y, \varphi_i \rangle|^2 \le B\|y\|^2, \quad \forall\, y \in \mathbb{R}^d.$$

2. For a frame $\Phi = \{\varphi_i\}_{i=1}^s$, the frame operator $S : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ is

$$S(y) = \sum_{i=1}^{s} \langle y, \varphi_i \rangle \varphi_i.$$

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Frames and HSI data

- We wish to use a data dependent frame $\Phi = \{\varphi_i\}_{i=1}^{s}$ to represent the reduced dimension space $Y = \{y_i\}_{i=1}^{N} \subset \mathbb{R}^d$:

$$y_i = \sum_{j=1}^{s} c_{i,j}\varphi_j, \quad \forall\, y_i \in Y.$$

- Possible frame construction algorithms:
  - existing endmember algorithms
  - modified frame potential [Benedetto, Fickus; 2003]
- Possible frame coefficients:
  - $c_{i,j} = \langle y_i, S^{-1}(\varphi_j)\rangle$
  - $c_{i,\cdot} = \arg\min_c \|c\|_{\ell^1}$ subject to $\Phi c = y_i$

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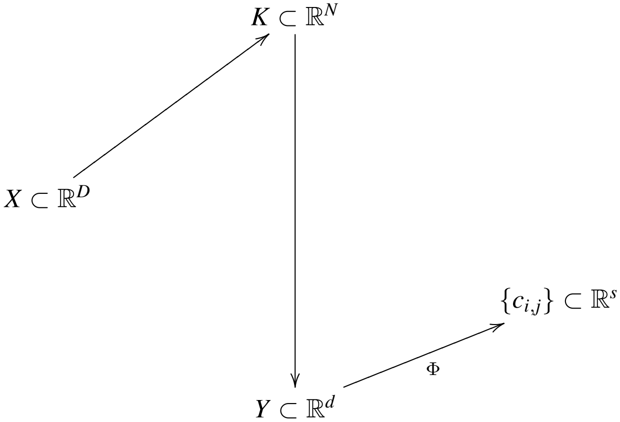
Why use frames?

Why use frames?

- Traditional endmembers may be mixtures of many prominent features.
- Overestimating the number of classes allows for flexibility in representing mixtures and pure elements.
- Empirical evidence suggests that distinct classes are not orthogonal to each other. Unlike the orthogonal eigenvectors of K, frame elements need not be orthogonal.

Review of algorithm



Outline

1. Hyperspectral imagery data
   - Introduction to hyperspectral imagery data
   - Endmembers
2. The algorithm
   - Kernel eigenmap methods
   - Frames
3. Results
   - The Urban data set
   - Classification results

Urban

Figure: HYDICE Copperas Cove, TX – http://www.tec.army.mil/Hypercube/

Urban classes

22 classes in the Urban data set, including:

- Dark asphalt
- Vegetation: grass
- Soil 1
- Soil 2
- Soil 3 (dark)
- Roof: Walmart

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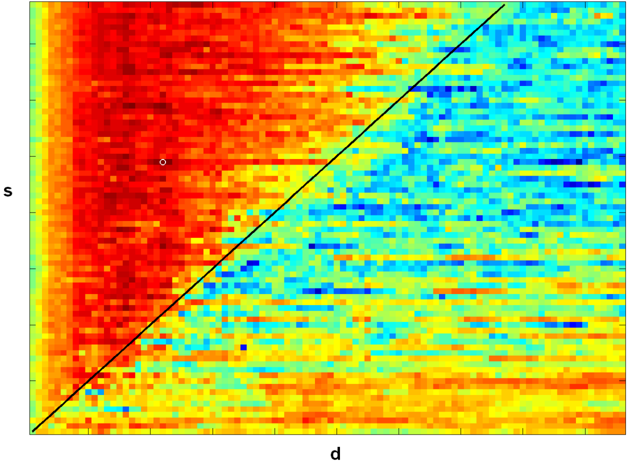
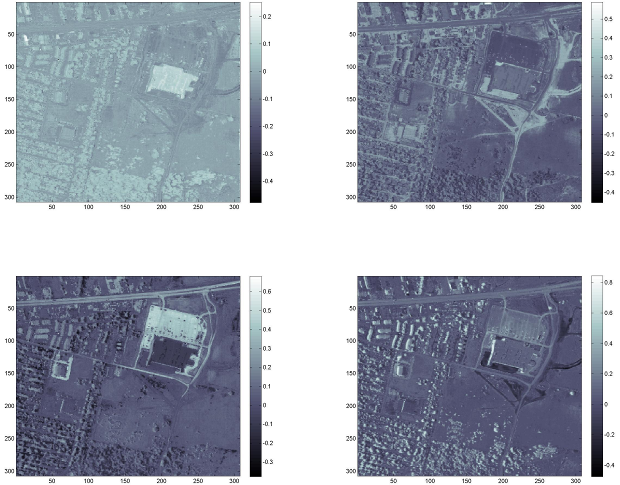
Overview of classification results

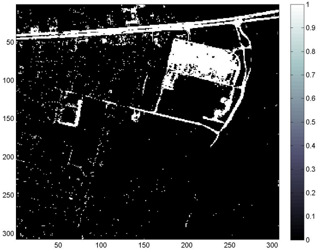


Sample frame coefficients



Dark asphalt

(e) Urban



(f) Dark Asphalt

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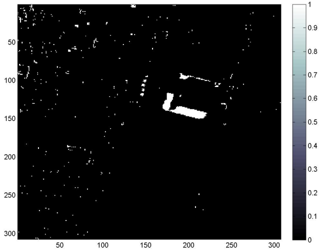
Light asphalt

(a) Urban

(b) Light asphalt

Concrete

(a) Urban



(b) Concrete

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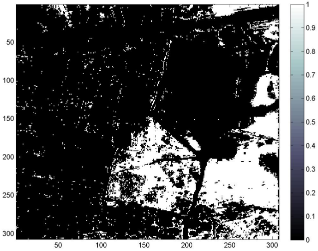
Vegetation: pasture

(a) Urban



(b) Vegetation: pasture

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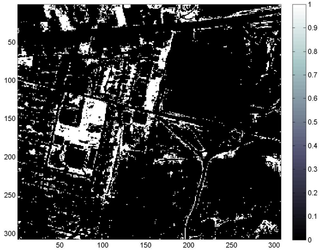
Vegetation: grass

(a) Urban



(b) Vegetation: grass

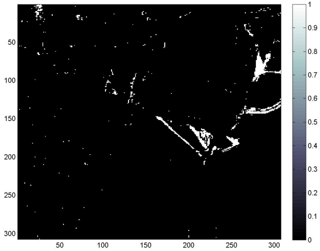
Vegetation: trees

(a) Urban

(b) Vegetation: trees

Soil 1

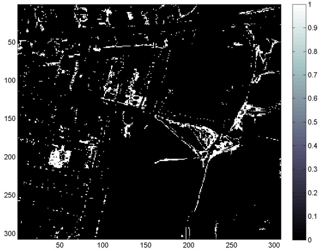
(a) Urban



(b) Soil 1

Soil 2

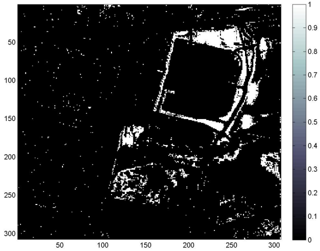
(a) Urban



(b) Soil 2

Soil 3 (dark)

(a) Urban



(b) Soil 3 (dark)

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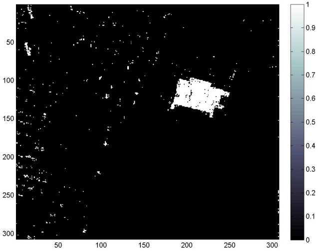
Roof: Walmart

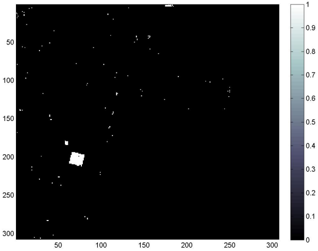
(a) Urban



(b) Roof: Walmart

Roof: A

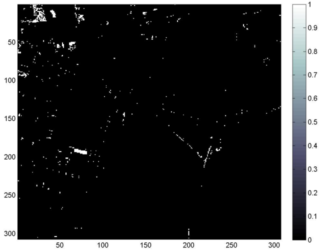
(a) Urban



(b) Roof: A

Roof: B

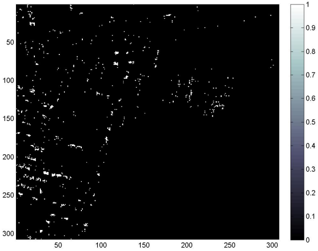
(a) Urban



(b) Roof: B

Roof: light gray

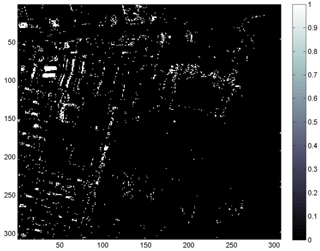
(a) Urban



(b) Roof: light gray

Roof: dark brown

(a) Urban



(b) Roof: dark brown

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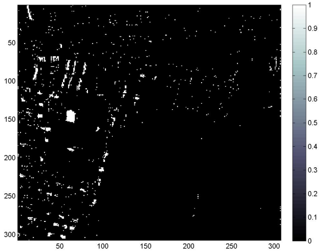
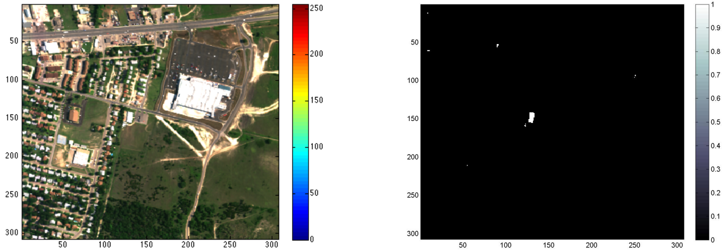
Roof: church

(a) Urban



(b) Roof: church

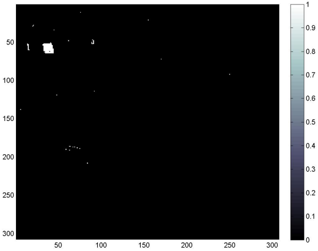
Roof: school



(a) Urban (b) Roof: school

Roof: bright

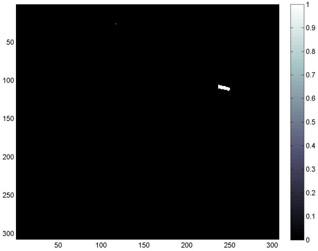
(a) Urban



(b) Roof: bright

Roof: blue green

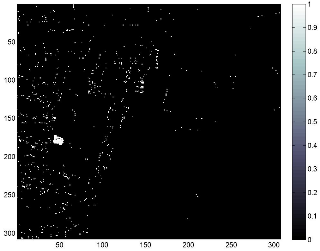
(a) Urban



(b) Roof: blue green

Tennis court

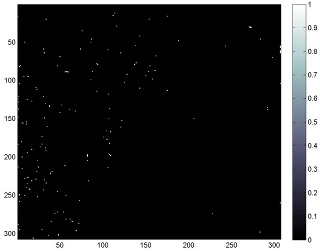
(a) Urban



(b) Tennis court

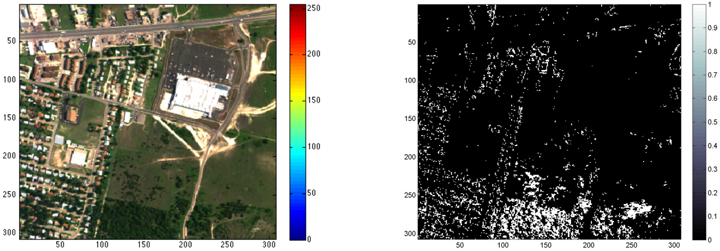
Pool water

(a) Urban



(b) Pool water

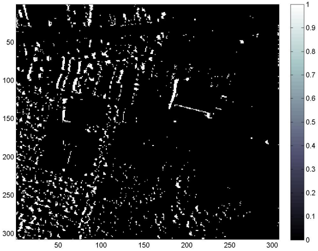
Shaded vegetation



(a) Urban (b) Shaded vegetation

Shaded pavement

(a) Urban



(b) Shaded pavement

Thank you

Thank you for your time.

http://www.math.umd.edu/∼hirn/